Mathematics



Grade 5 Module 2

Grade 5 • Module 2

Multi-Digit Whole Number and Decimal Fraction Operations

OVERVIEW

In Module 1, students explored the relationships of adjacent units on the place value chart to generalize whole number algorithms to decimal fraction operations. In Module 2, students apply the patterns of the base ten system to mental strategies and the multiplication and division algorithms.

Topics A through D provide a sequential study of multiplication. To link to prior learning and set the foundation for understanding the standard multiplication algorithm, students begin at the concrete—pictorial level in Topic A. They use number disks to model multi-digit multiplication of place value units, e.g., 42×10 , 42×100 , $42 \times 1,000$, leading to problems such as 42×30 , 42×300 and $42 \times 3,000$. They then round factors in Lesson 2 and discuss the reasonableness of their products. Throughout Topic A, students evaluate and write simple expressions to record their calculations using the associative property and parentheses to record the relevant order of calculations.

In Topic B, place value understanding moves toward understanding the distributive property via area diagrams which are used to generate and record the partial products of the standard algorithm. Topic C moves students from whole numbers to multiplication with decimals, again using place value as a guide to reason and make estimations about products. In Topic D, students explore multiplication as a method for expressing equivalent measures. For example, they multiply to convert between meters and centimeters or ounces and cups with measurements in both whole number and decimal form.

Topics E through H provide a similar sequence for division. Topic E begins concretely with number disks as an Introduction to division with multi-digit whole numbers. In the same lesson, $420 \div 60$ is interpreted as $420 \div 10 \div 6$. Next, students round dividends and two-digit divisors to nearby multiples of 10 in order to estimate single-digit quotients (e.g., $431 \div 58 \approx 420 \div 60 = 7$) and then multi-digit quotients. This work is done horizontally, outside the context of the written vertical method.

The series of lessons in Topic F leads students to divide multi-digit dividends by two-digit divisors using the written vertical method. Each lesson moves to a new level of difficulty with a sequence beginning with divisors that are multiples of 10 to non-multiples of 10. Two instructional days are devoted to single-digit quotients with and without remainders before progressing into two- and three-digit quotients.

In Topic G, students use their understanding to divide decimals by two-digit divisors in a sequence similar to that of Topic F with whole numbers.

In Topic H, students apply the work of the module to solve multi-step word problems using multi-digit division with unknowns representing either the group size or number of groups. In this topic, an emphasis on checking the reasonableness of their answers draws on skills learned throughout the module, including refining their knowledge of place value, rounding, and estimation.

**The sample questions/responses contained in this manual are straight from http://www.engageny.org/. They are provided to give some insight into the kinds of skills expected of students as the lesson is taught.

Terminology

New or Recently Introduced Terms

- Decimal Fraction (a proper fraction whose denominator is a power of 10)
- Multiplier (a quantity by which a given number—a multiplicand—is to be multiplied)
- Parentheses (the symbols used to relate order of operations)

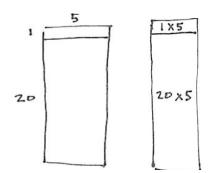
Familiar Terms and Symbols

- Decimal (a fraction whose denominator is a power of ten and whose numerator is expressed by figures placed to the right of a decimal point)
- Digit (a numeral between 0 and 9)
- Divisor (the number by which another number is divided)
- Equation (a statement that the values of two mathematical expressions are equal)
- Equivalence (a state of being equal or equivalent)
- Equivalent measures (e.g., 12 inches = 1 foot; 16 ounces = 1 pound)
- Estimate (approximation of the value of a quantity or number)
- Exponent (the number of times a number is to be used as a factor in a multiplication expression)
- Multiple (a number that can be divided by another number without a remainder like 15, 20, or any multiple of 5)
- Pattern (a systematically consistent and recurring trait within a sequence)
- Product (the result of a multiplication of factors)
- Quotient (the answer of dividing one quantity by another)
- Remainder (the number left over when one integer is divided by another)
- Renaming (making a larger unit)
- Rounding (approximating the value of a given number)
- Unit Form (place value counting, e.g., 34 stated as 3 tens 4 ones)

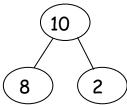
Suggested Tools and Representations

- Area models (e.g., an array)
- Number bond (visual model which shows a larger section as the sum of the smaller sections or parts)
- Number disks (visual representation of a digit's value by place value disks, e.g., 34 = 10, 10, 10, 1, 1, 1, 1 in disks)
- Partial product (an algorithmic method that takes base ten decompositions of factors, makes products of all pairs, and adds all products together)
- Partial quotient (an algorithmic method using successive approximation)

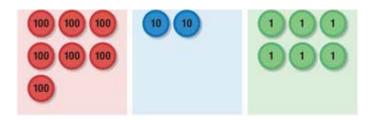
Area Model: These area models both show a visual representation of 21 x 5, breaking apart the 21 by tens and ones (20 and 1).



Number Bond: This bond is showing that the addends or parts, 8 and 2, together total the sum, 10.



Number Disks: The value 726 is shown using number disks.



Partial Products: The partial products, 23 and 690, are added to get the final product of 713.

Objective: Multiply multi-digit whole numbers and multiples of 10 using place value patterns and the distributive and associative properties.

a.
$$23 \times 20$$

Think: 23 ones × 2 tens =
$$46$$
 tens = $3 \times 20 = 460$

c.
$$40 \times 5$$
 40×50 40×500 40×5000 $= (4 \times 100) \times (5 \times 100) = (4 \times 5) \times (100 \times 1000)$ $= (4 \times 5) \times (100 \times 1000) = (4 \times 5) \times (100 \times 1000)$ $= (4 \times 5) \times (100 \times 1000) = (20 \times 100000)$ $= 20 \times 1000000$ $= 20 \times 1000000$

Lesson 2

Objective: Estimate multi-digit products by rounding factors to a basic fact and using place value patterns.

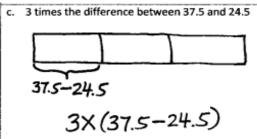
1. Round the factors to estimate the products.

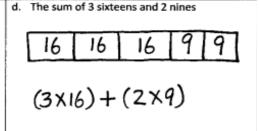
A reasonable estimate for 597 × 52 is 30,000

- Michael saves \$423 dollars a month for college.
 - a. About how much money would he have saved after 4 years?

$$4\times12=48$$
 months
 $$423\times48\approx$400\times50=$20,000$
He would have saved about \$20,000 after 4 years.

Objective: Write and interpret numerical expressions and compare expressions using a visual model.





- 6. A box contains 24 oranges. Mr. Lee ordered 8 boxes for his store and 12 boxes for his restaurant,
 - a. Write an expression to show how to find the total number of oranges ordered.

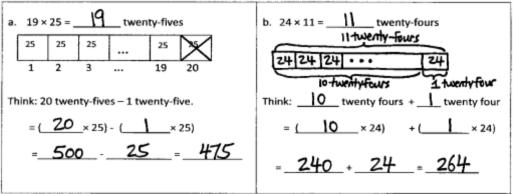
b. Next week, Mr. Lee will both double the number of boxes he orders. Write a new expression to represent the number of oranges in next week's order.

Lesson 4

Objective: Convert numerical expressions into unit form as a mental strategy for multi-digit multiplication.

5. Solve mentally.

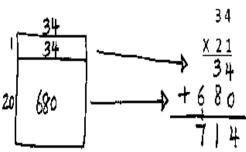
Solve using mental math. Draw a tape diagram and fill in the blanks to show your thinking. The first one was done for you.



Objective: Connect visual models and the distributive property to partial products of the standard algorithm without renaming.

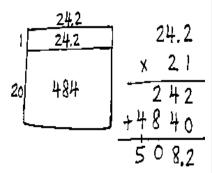
- 1. Draw an area model and then solve using the standard algorithm. Use arrows to match the partial products from the area model to the partial products of the algorithm.
- 4. Farmer Brown feeds 12.1 kg of alfalfa to each of his 2 horses daily. How many kilograms of alfalfa will all his horses have eaten after 21 days? Draw an area model to solve.





34 | 12.1 + 12.1 = 24.2 kg

$$\Rightarrow \frac{x \cdot 21}{34}$$
 | 1 unit = 24.2 kg
 $\Rightarrow \frac{x \cdot 21}{34}$ | 21 units = 24.2 x 21
 $\Rightarrow \frac{x \cdot 21}{34}$ = 508.2 kg

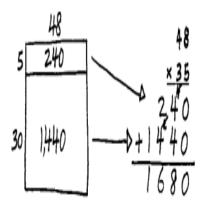


Lesson 6

Objective: Connect area diagrams and the distributive property to partial products of the standard algorithm without renaming.

1. Draw an area model, and then solve using the standard algorithm. Use arrows to match the partial products from your area model to the partial products in the algorithm.

a. 48 × 35



4. General admission to The American Museum of Natural History is \$19. a. If a group of 125 students visits the museum, how much will the

group's tickets cost? 1 unit = \$19 125
125 units = 125 × 19 = \$2,375
$$\times$$
 19
1125
+ 1250
2375
The group's tickets will cost \$2,375.

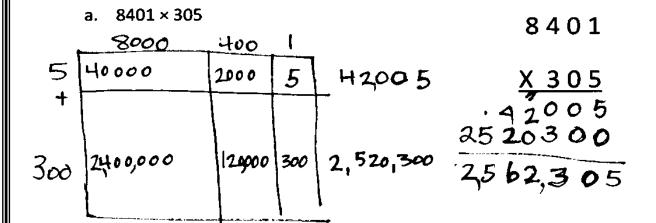
b. If the group also purchases IMAX movie tickets for an additional \$4 per student, what is the new total cost of all the tickets? Write an expression that shows how you calculated the new price.

$$(19+4) \times 12S = 23 \times 12S = 2,87S = 2,87S = 12S = 2,87S$$

The new total cost of all the tickets will be \$2,875.

Objective: Connect area diagrams and the distributive property to partial products of the standard algorithm with renaming.

2. Solve by drawing the area model and using the standard algorithm.



Lesson 8

Objective: Fluently multiply multi-digit whole numbers using the standard algorithm and using estimation to check for reasonableness of the product.

1. Estimate the product first. Solve by using the standard algorithm. Use your estimate to check the reasonableness

of the product.	a. 213 × 328 ≈ 200 × 300 =60,000 +6	213 × 328 1704 4260 3900 9,864	= Z80,000 + 1 4	662 × 372 1324 46340 18600 -6,264	280,000 = 280,000 24 24 - 24 - 326	739 × 442 1478 9560 5638
					54	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,

2. Each container holds 1 L 275 mL of water. How much water is in 609 identical containers? Find the difference

between your estimated product and precise product.

Estimate: 1200 ml
$$\times$$
 600 Actual: 1275 ml \times 609
$$= 720,000 ml$$

$$= 720 L$$

$$= 776 L 475 ml$$

$$= 776 L 475 ml$$

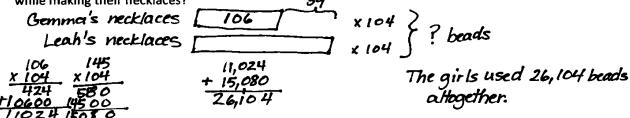
$$= 776 L 475 ml$$

My actual product was 561 475ml larger than the estimated product.

Objective: Fluently multiply multi-digit whole numbers using the standard algorithm to solve multi-step word problems.

- 2. Gemma and Leah are both jewelry makers. Gemma made 106 beaded necklaces. Leah made 39 more necklaces than Gemma.
 - a. Each necklace they make has exactly 104 beads on it. How many beads did both girls use altogether while making their necklaces?

 39



b. At a recent craft fair, Gemma sold each of her necklaces for \$14. Leah sold each of her necklaces for 10 dollars more. Who made more money at the craft fair? How much more?

Leah made \$1,996 more.

Leah made \$1,996 more.

$$\frac{106}{x} \frac{145}{1484} = \frac{24}{580} = \frac{1484}{51996}$$

Lesson 10

Objective: Multiply decimal fractions with tenths by multi-digit whole numbers using place value understanding to record partial products.

1. Estimate the product. Solve using an area model and the standard algorithm. Remember to express your products in standard form. $22 \times 2.4 \approx 20 \times 2.4 \approx 2.4 \times 2.4 \approx 2.4 \times 2.4 \approx 2.4 \times 2.4 \approx 2.4 \times 2.4 \times 2.4 \approx 2.4 \times 2.4$



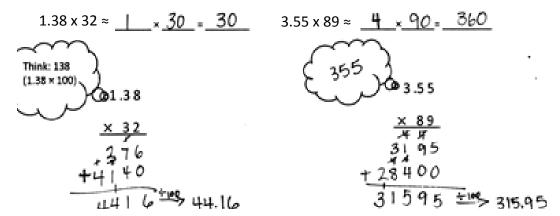
1. Estimate, then use the standard algorithm to solve. Express your products in standard form.

$$3.2 \times 47 \approx \underline{3} \times \underline{50} = \underline{/50}$$
 $3.2 \times 94 \approx \underline{3} \times \underline{90} = \underline{270}$
 3.2 (tenths) 3.2 (tenths)
 $\frac{\times 4.7}{2.2.4}$
 $+|2.6.0|$ $\frac{\times 94}{|2.8.0|}$
 $+|2.6.0|$ $\frac{\times 94}{|2.8.0|}$

Objective: Multiply decimal fractions by multi-digit whole numbers through conversion to a whole number problem and reasoning about the placement of the decimal.

1. Estimate the product. Solve using the standard algorithm. Use the thought bubbles to show your

thinking. (Draw an area model on a separate sheet if it helps you.)

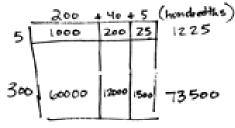


- 3. Use the whole number product and place value reasoning to place the decimal point in the second product. Explain how you know.
 - a. If 98 × 768 = 75,264 then 98 × 7.68 = 752.64
 - 7.68 would be like 768 (hundredths) so just divide by 100

Lesson 12

Objective: Reason about the product of a whole number and a decimal with hundredths using place value understanding and estimation.

1. Estimate, and then solve using the standard algorithm. You may draw an area model if it helps you.



- 2. Estimate, and then solve using the standard algorithm. You may draw an area model if it helps you.
- a. 1.23×12 分 | ¥ 12. ← 12.

Objective: Use whole number multiplication to express equivalent measurements.

- 4. Convert. Use your Reference Sheet to remind you of the conversion factors. Show your work.
- a. 27 ft = 324 in
- d. 7 kg = 7000 g
- g. 3 km 85 m = 3,085 m

- 300+85
- 6. Ben helps his dad make chicken soup. Their recipe makes 15 cups of soup. If they each eat 2 cups and freeze the rest, will the leftovers fit in a 64-ounce container?

Lesson 14

Objective: Use decimal multiplication to express equivalent measurements.

1. Convert. Use your reference sheet to help you remember the conversion factors.

3. Emma can't believe how huge the Statue of Liberty is. She finds more information about Lady Liberty. Help Emma fill in the rest of the chart and then answer the questions.

The Statue of	CUSTOM	ARY UNITS	METRIC UNITS	
Liberty's	Feet	Inches	Meters	Centimeters
nose	48+6 4 ft 6 in	54 in	1.37 m	137cm
Index finger	8 ft × 12	96 in	2.44 m	244cm
head	17ft 3 in	207 in	5.26 m	526cm
eye	2 ft 6 in	30 in	0.76 m	76 cm

a. Emma is 52 inches tall. Which of Lady Liberty's body parts above is the closest to Emma's height? What is the difference between these two measurements in inches?

The mose is the closest. It is 2 inches longer.

Objective: Solve two-step word problems involving measurement and multi-digit multiplication.

- 3. Josie is 1.4 meters tall. Her sister is 54 cm shorter.
 - a. Find her sister's height in meters.

Josu's sister is 0.86m tall.

b. How tall are Josie and her sister combined, in meters?

The sisters are 2.26m tall altogether.

Lesson 16

Objective: Use divide by 10 patterns for multi-digit whole number division.

 Divide. Draw number disks to show your thinking for (a) and (c). You may draw disks on your white board to solve the others if necessary.

- 3. The floor of a rectangular banquet hall has an area of 3600 m^2 . The length is 90 m.
 - a. What's the width of the banquet hall?

The width of the banquet hall was 40m.

Objective: Use basic facts to estimate quotients with two-digit divisors.

1. Estimate the quotient for the following problems. Round the divisor first.

 A video game store has a budget of \$825 and would like to purchase new video games. If each video game costs \$41, estimate the total number of video games the store can purchase with their budget. Explain your thinking.

I estimated \$825 divided by \$41 to be \$800 \times 40, and got 20. This means that the store can purchase a total of 20 video games with their budget.

Lesson 18

Objective: Use basic facts to estimate quotients with two-digit divisors.

Estimate the quotient for the following problems. The first one is done for you.

Janice bought 28 apps for her phone that, altogether used 1348 MB of space.

a. If each app used the same amount of space, about how many MB of memory did each app use?
Show how you estimated.

Each app used about 40 MB of momory.

Objective: Divide two- and three-digit dividends by multiples of 10 with singledigit quotients and make connections to a written method.

Divide, then check. The first one is done for you.

a. 41 ÷ 30

Check:

$$30 + 11 = 41$$

A number divided by 80 has a quotient of 7 with 4 as a remainder. Find the number.

The number was 564.

Lesson 20

Objective: Divide two- and three-digit dividends by two-digit divisors with singledigit quotients and make connections to a written method.

1. Divide, then check with multiplication. The first one is done for you.

$$\begin{array}{rrr}
2 & R20 \\
32 \sqrt{84} & 32 \times 2 = 64 \\
-64 & 64 + 20 = 84
\end{array}$$

5. Mrs. Silverstein sold 91 cupcakes at a food fair. The cupcakes were sold in boxes of "a baker's dozen," which is 13. She sold all the cupcakes at \$15 per box. How much money did she receive?

Objective: Divide two- and three-digit dividends by two-digit divisors with single-digit quotients and make connections to a written method.

1. Divide, then check using multiplication. The first one is done for you.

Check:

3. Assume that Mrs. Giang's car travels 14 miles on each gallon of gas. If she travels to visit her niece who lives 133 miles away, how many gallons of gas will Mrs. Giang need to make the round trip?

Lesson 22

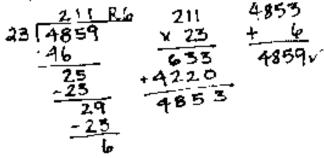
Objective: Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients, reasoning about the decomposition of successive remainders in each place value.

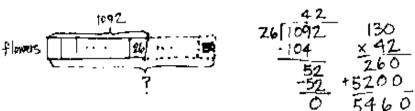
 Halle solved 664 ÷ 48 below, and got a quotient of 13 remainder 40. How could she use her work below to solve 659 ÷ 48 without redoing the work? Explain your thinking.

Since the whole of 659 is 5 less than the original whole of 664. It means that instead of a remainder of 40, it should be 35. The quotient of 659 divided by 48 is 13 with a remainder of 35.

Objective: Divide three- and four-digit dividends by two-digit divisors resulting in two- and three-digit quotients, reasoning about the decomposition of successive remainders in each place value.

- 1. Divide, then check using multiplication.
- a. $4,859 \div 23$
- 3. 1,092 flowers are arranged into 26 vases, with the same number of flowers in each vase. How many flowers would be needed to fill 130 such vases?





5,460 flowers would be needed to fill 130 vases.

Lesson 24

Objective: Divide decimal dividends by multiples of 10, reasoning about the placement of the decimal point and making connections to a written method.

1. Divide. Show the division in the right hand column in two steps. The first two have been done for you.

a.
$$1.2 \div 6 = 0.2$$

b.
$$1.2 \div 60 = (1.2 \div 6) \div 10 = 0.2 \div 10 = 0.02$$

$$2.4 \div 40 = \frac{2.4 \div 4}{10} \div 10 = 0.6 \div 10 = 0.06$$

Use place value reasoning and the first quotient to compute the second quotient. Explain your thought process.

a.
$$46.5 \div 5 = 9.3$$

Objective: Use basic facts to approximate decimal quotients with two-digit divisors, reasoning about the placement of the decimal point.

- 2. Estimate the quotient in (a). Use your estimated quotient to estimate (b) and (c). 3. Edward bikes the same route to and from school each day. After 28 school days, he bikes a total distance of 389.2 miles.
 - a. $7.16 \div 36 \approx 8 \div 40 = (8 \div 4) \div 10 = 2 \div 10 = 0.2$
- a. Estimate how many miles he bikes in one day.

b. 716÷36≈ 800÷40 = 20

Edward bites about 13 miles a day

c. $71.6 \div 36 \approx 80 \div 40 = 2$

b. If Edward continues his routine of biking to school, about how days altogether will it take him to reach a total distance of 500 miles?

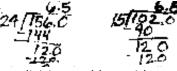
$$500 \div 13$$

 $\approx 450 \div 15 = 30$ It will take about 40 days
 $\approx 480 \div 12 = 40$ to reach 500 miles

Lesson 26

Objective: Divide decimal dividends by two-digit divisors, estimating quotients, reasoning about the placement of the decimal point, and making connections to a written method.

1. 156 ÷ 24 and 102 ÷ 15 both have a quotient of 6 and a remainder of 12. a. Are the division expressions equivalent to each other? Use your knowledge of decimal division to justify your answer.



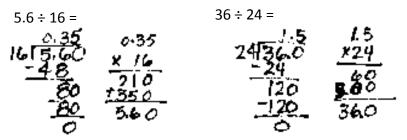
b. Construct your own division problem with a two-digit divisor that has a quotient of 6 and a remainder of 12 17×6=60+42=102 + 12 -114 but is not equivalent to the problems in 1(a).

114+17

3. The weight of 72 identical marbles is 183.6 grams. What is the weight of each marble? Explain how you know the decimal point of your quotient is placed reasonably.

> Each marble weighs 2.55 grams. This makes sense because 183.6+72 is about 180760 which is 3.

Objective: Divide decimal dividends by two-digit divisors, estimating quotients, reasoning about the placement of the decimal point, and making connections to a written method. 1. Divide. Check your work with multiplication.



4. A soccer coach spent \$162 dollars on 24 pairs of socks for his players. How much did five pairs of socks cost?

Lesson 28

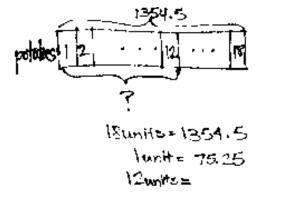
Objective: Solve division word problems involving multi-digit division with group size unknown and the number of groups unknown.

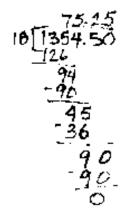
3. Jim Nasium is building a tree house for his two daughters. He cuts 12 pieces of wood from a board that is 128 inches long. He cuts 5 pieces that measure 15.75 inches each, and 7 pieces evenly cut from what is left. Jim calculates that due to the width of his cutting blade, he will lose a total of 2 inches of wood after making all of the cuts. What is the length of each of the seven pieces?

Each of the 7 pieces 15 6.75 inches long.

Objective: Solve division word problems involving multi-digit division with group size unknown and the number of groups unknown.

1. Lamar has 1,354.5 kilograms of potatoes to deliver equally to 18 stores. 12 of the stores are in the Bronx. How many kilograms of potatoes will be delivered to stores in the Bronx?

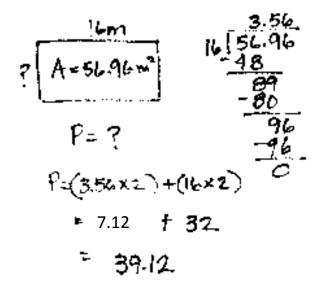






903 pounds of potaties Will be delivered to the Bronx.

3. The area of a rectangle is 56.96 m₂. If the length is 16 m, what is its perimeter?



The perimeter of the rectangle is 39.12 m.